

Logic Gates

Department: Mathematical and Computing Science

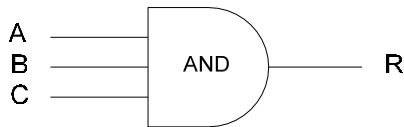
Course Code: SWE 204

Course Title: Logic and Its Applications in Computer Science

Logic gates are the mechanism used to convert Boolean logic into the circuitry the computer needs to solve such problems.

We have learned about three(3) different gates.

The AND Gate takes two or more inputs. Remember that the AND operator examines all the inputs. If they are all True (1) the result is True (1). If any of the inputs are False(0) the Result is False(0)



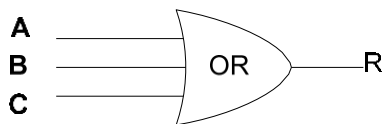
The equation for this AND gate is:

$$R = ABC$$

The Truth Table for this AND gate is:

A	B	C	ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

The OR Gate takes two or more inputs. Remember that the OR operator examines all the inputs. If any of the inputs are True(1) the Result is True(1).



The equation for this OR gate is:

$$R = A+B+C$$

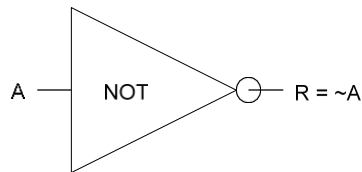
The Truth Table for this OR gate is:

A	B	C	A+B+C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

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The last gate we have discussed is the NOT gate and this gate simply inverts whatever data enters it. If the input is True(1) the output is False(0). If the input is False(0) the output is True(1).

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The equation for this NOT gate is:

$$R = \sim A$$

The Truth Table for this NOT gate is:

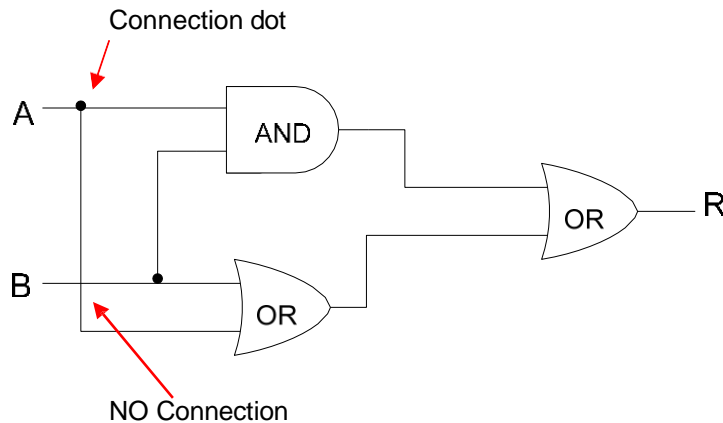
A	$\sim A$
0	1
1	0

These are the building blocks of computer circuitry. By combining them in the correct order all of the fancier circuitry is build.

Logic Networks

Gates are interconnected together to form circuits that perform more complex functions care called Logic Networks.

- Wires connect gates
- When two wires cross a dot represents a connection



This network represents the following Boolean statement. $R = AB + (A+B)$

How do I know this?

Examining the diagram we can assume that if the inputs are on the left, and the output on the right then the “current” in this circuit moves from left to right.

What do we encounter first?

A AND B enters at the top (AB)
 B OR A enters at the bottom (B+A)

So thus far $R \rightarrow AB \quad (B+A)$

The outputs from both of these gates goes into an OR gate

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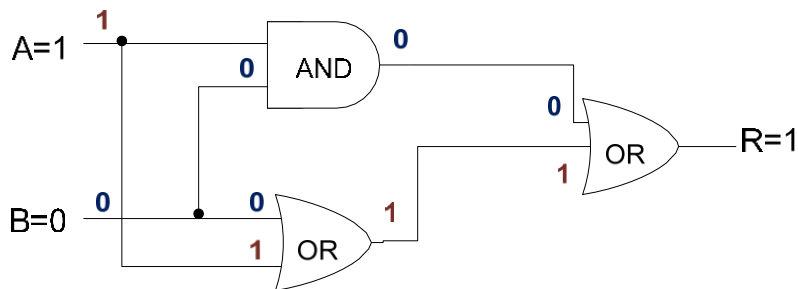
So $\underbrace{AB \quad (B+A)}_{\text{OR}(+)}$ $\rightarrow R = AB + (B+A)$

The Truth Table is:

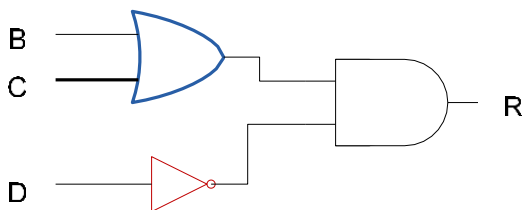
A	B	AB	(A+B)	AB+(A+B)
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	1

Using the Truth Table above, If $A = 1$, $B = 0$, What is R ? We can read across the third line of the table and see that $R = 1$.

There is another way to find this value. In the diagram below the values are placed on the gate diagram. The input to the gate is identified, then the output is identified and we can determine R without using a truth table. The output from the final OR gate $R = 1$.



Example 1: What is logic equation represented by this circuit?



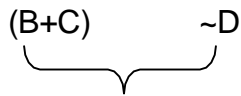
There are three gates used in this example. B and C are input to the top OR gate. D enters a NOT gate. The output from the OR gate and the NOT gate become the input to the final AND gate.

When data is input to the gates on the left, what do we know

B, C go into the OR gate $(B+C)$

D goes into the NOT gate $\sim D$

Logic Gates



AND(\cdot)

$$R = (B+C) \cdot \sim D$$

The Truth Table is:

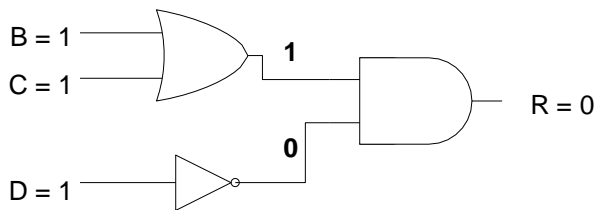
$$R = (B+C) \cdot \sim D$$

If $B = 1$, $C = 1$, $D = 1$, What is R ?

Reading across the last line, we can see that $R = 0$

B	C	D	(B+C)	$\sim D$	(B+C) \cdot $\sim D$
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	0	0
1	1	0	1	1	1
1	1	1	1	0	0

Another way to solve this problem is by placing values directly on the circuit diagram. The output from each gate is then determined by the input into it.



Example 2:

This is an expansion of the above circuit.

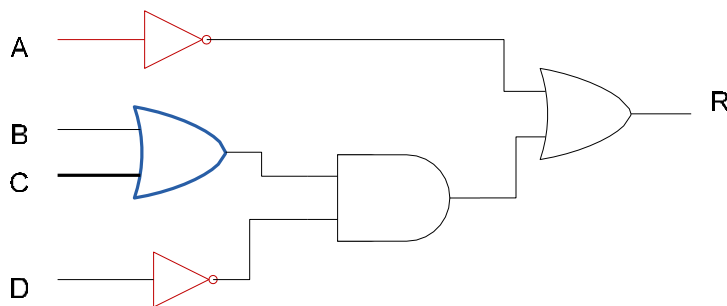
What equation does the following circuit represent?

Construct the truth table for the following circuit.

If $A = 1$, $B = 1$, $C = 0$ and $D = 0$, what is R ?

If $A = 0$, $B = 0$, $C = 1$ and $D = 1$, what is R ?

Look carefully at the component parts of this diagram.



The topmost NOT gate takes A as input. Its output is $\sim A$. Below that is an OR gate which takes both B and C as input. Its output is contained in the expression $(B+C)$. At the bottom in another NOT gate which can be represented as $\sim D$.

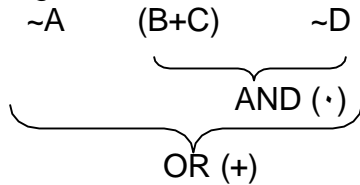
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So far we have

$\sim A$ $(B+C)$ $\sim D$

The output of the OR gate and the NOT gate are input to an AND gate.

The output of this AND gate is input to the final OR gate whose other input is the topmost NOT gate.



So: $R = \sim A + ((B+C)(\sim D))$ or $R = \sim A + ((B+C) \cdot \sim D)$

Truth Table for $R = \sim A + ((B+C) \cdot \sim D)$

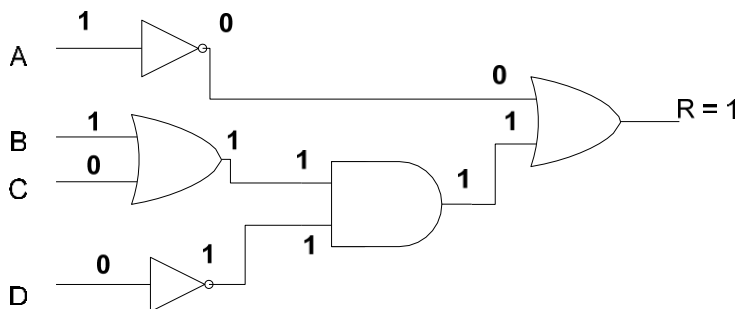
A	B	C	D	$(B+C)$	$\sim D$	$(B+C)(\sim D)$	$\sim A + ((B+C)(\sim D))$
0	0	0	0	0	1	0	1
0	0	0	1	1	0	0	1
0	0	1	0	1	1	1	1
0	0	1	1	1	0	0	1
0	1	0	0	1	1	1	1
0	1	0	1	1	0	0	1
0	1	1	0	1	1	1	1
0	1	1	1	1	0	0	1
1	0	0	0	0	1	0	0
1	0	0	1	0	0	0	0
1	0	1	0	0	1	1	1
1	0	1	1	0	1	0	0
1	1	0	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	0	0	1	1	1
1	1	1	1	0	0	0	0

If $A = 1, B = 1, C = 0$ and $D = 0$, what is R ? Using the Truth Table $R = 1$

If $A = 0, B = 0, C = 1$ and $D = 1$, what is R ? Using the Truth Table $R = 1$

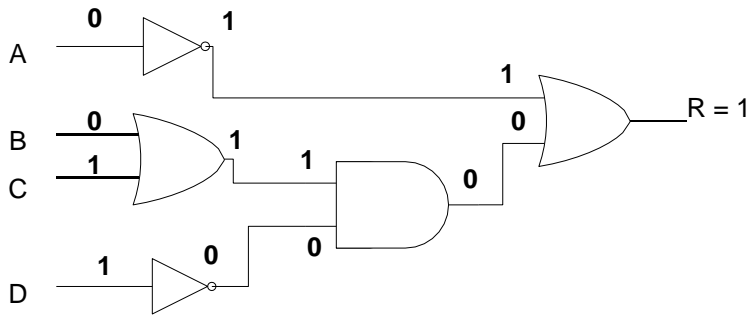
As with the previous example we can use the circuit diagram directly to determine the value of R for each question.

If $A = 1, B = 1, C = 0$ and $D = 0$, what is R ? Using the circuit $R = 1$



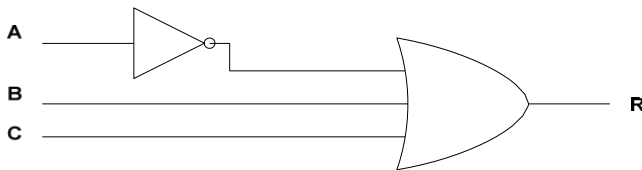
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Similarly, If $A = 0$, $B = 0$, $C = 1$ and $D = 1$, what is R ? Using the circuit $R = 1$

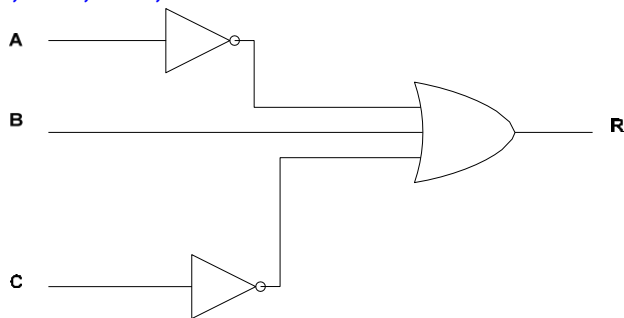


Try These Examples:

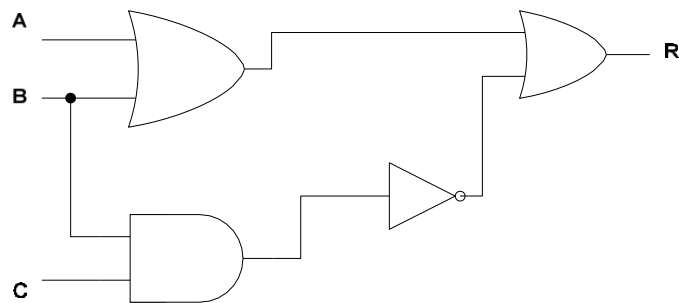
1) If $A = 0$, $B = 1$, $C = 0$, what is R ?



2) If $A = 1$, $B = 0$, $C = 1$, what is R ?

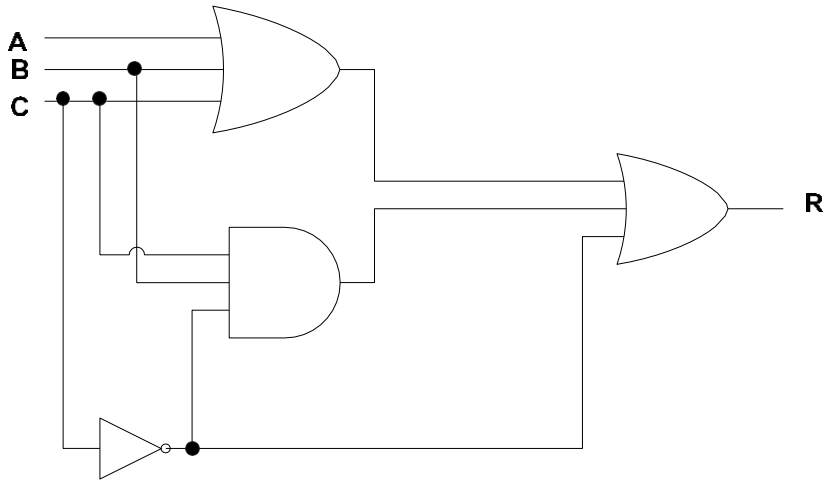


3) If $A = 1$, $B = 0$, $C = 1$, What is R ?



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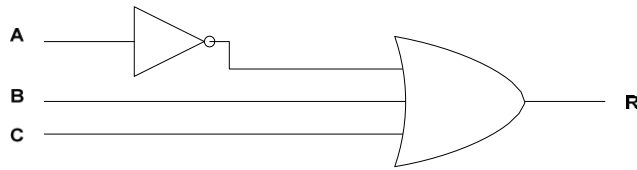
4) If $A = 0$, $B = 1$, $C = 1$, What is R ?



Logic Gates

Solutions:

1) If $A = 0$, $B = 1$, $C = 0$, what is R ?

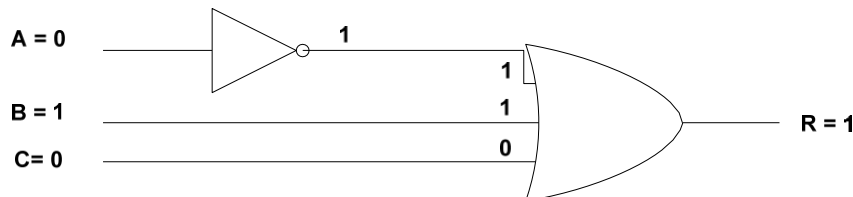


Reading from left to right (and top to bottom), we encounter $\sim A$ whose output along with B and C are all ORed together.

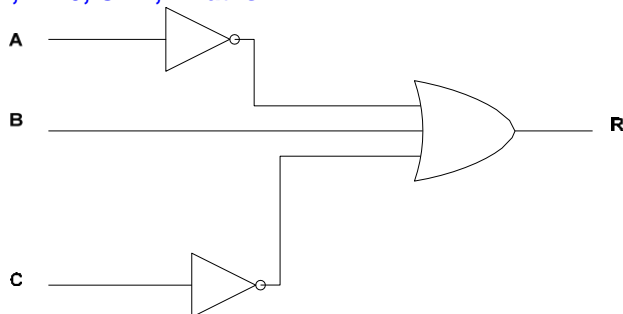
$R = \sim A + B + C$ this can also be written with parenthesis but they are not needed. $R = (\sim A + B + C)$

A	B	C	$\sim A$	$\sim A + B + C$
0	0	0	1	1
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$R = 1$



2) If $A = 1$, $B = 0$, $C = 1$, what is R ?



Reading from left to right and top to bottom we encounter

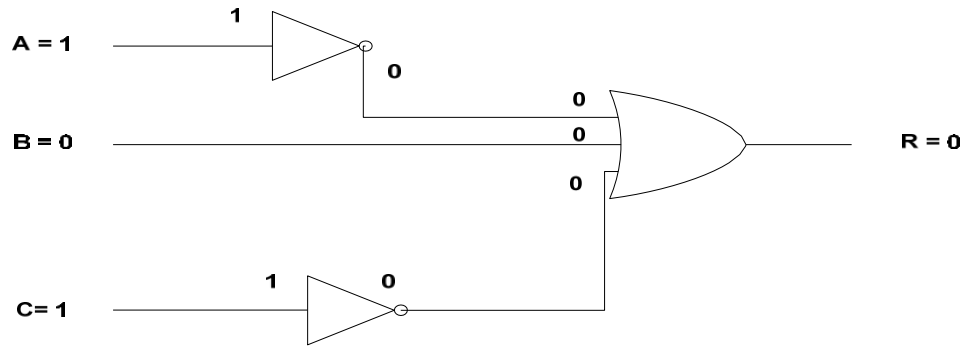
$$\underbrace{\sim A \quad B \quad \sim C}_{\text{OR}}$$

$$R = \sim A + B + \sim C$$

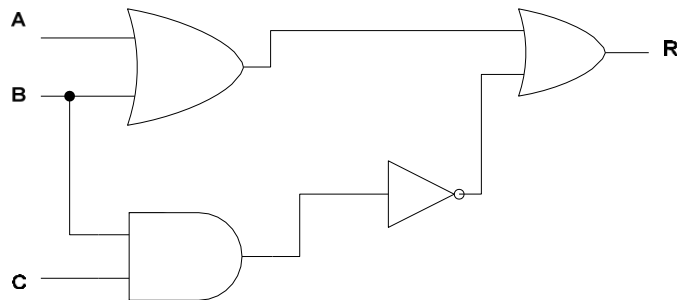
Reading the Truth Table below we see that $R = 0$

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A	B	C	$\sim A$	$\sim C$	$\sim A+B+\sim C$
0	0	0	1	1	1
0	0	1	1	0	1
0	1	0	1	1	1
0	1	1	1	0	1
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	0	1	1
1	1	1	0	0	1

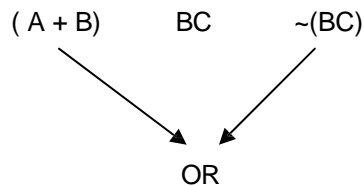


3) If $A = 1$, $B = 0$, $C = 1$, What is R ?



Looking at the circuit diagram from left to right and top to bottom, we see the following:

the input to the final gate is $(A+B)$ along with the result of $\sim(BC)$



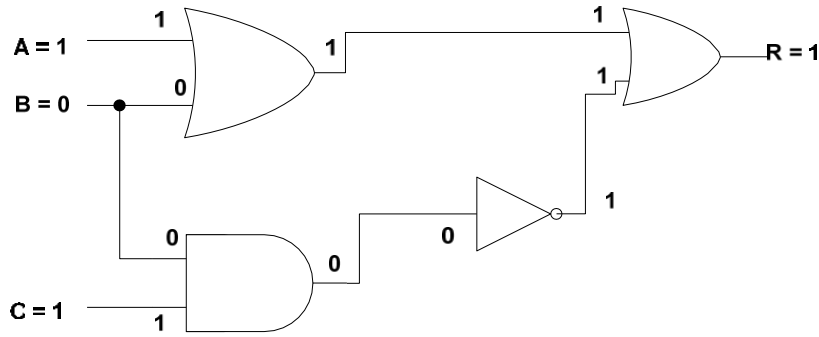
Note: the parenthesis must be around the BC because the result of this AND is inputted to the NOT gate.

$$R = (A+B) \sim(BC)$$

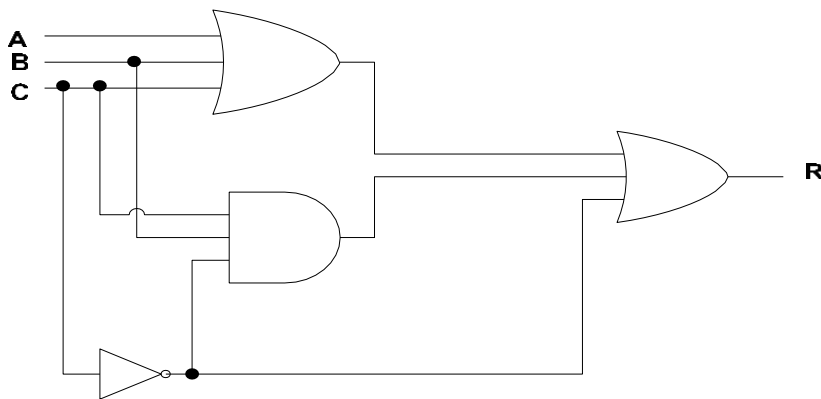
The Truth Table is shows that $R = 1$

A	B	C	BC	$\sim BC$	$(A+B)$	$(A+B) + \sim(BC)$
0	0	0	0	1	0	1
0	0	1	0	1	0	1
0	1	0	0	1	1	1
0	1	1	1	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	0	1	1	1
1	1	1	1	0	1	1

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4) If A = 0, B = 1, C = 1, What is R?



Reading the circuit from left to right and top to bottom,
 $(A+B+C)$ $\sim C$ $C \cdot B \cdot (\sim C)$ While this last expression may look odd it is "legal".



OR

The output from each of these expressions is input to the final OR gate.

$$R = (A+B+C) + (C \cdot B \cdot (\sim C)) + \sim C$$

Using the Truth Table method we learn that R = 1

A	B	C	$\sim C$	$(A+B+C)$	$C \cdot B \cdot (\sim C)$	$(A+B+C) + (C \cdot B \cdot (\sim C)) + \sim C$
0	0	0	1	0	0	1
0	0	1	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	1	0	1	1	0	1
1	1	1	0	1	0	1

Logic Gates

Using the Truth Table above we see that $R = 1$

Alternatively we could input the data directly into the circuit diagram and see the same results.

